CLAIMS

What is claimed is:

1. A computer program product, used in a computer system having a processor, for modeling an electromagnetic response of an arbitrarily shaped three-dimensional object to an arbitrary time-harmonic incident field by means of a well-conditioned boundary integral equation (BIE), the computer program product comprising:
   a. a recording medium;
   b. means, recorded on the recording medium for producing a discretized representation of a well-conditioned BIE to provide a well-conditioned, finite-dimensional linear system whose solution approximates a distribution of equivalent surface sources on a model caused by the arbitrary time-harmonic incident field; and
   c. means, recorded on the recording medium, for solving the linear system to determine values of the equivalent surface sources; whereby solution of the well-conditioned finite-dimensional linear system provides improved computational efficiency and solution accuracy.

2. A computer program product as set forth in claim 1, wherein the computer program product further comprises means, recorded on the recording medium for modeling electromagnetic scattering from the three-dimensional object due to the time-harmonic incident field as determined from the equivalent surface sources.

3. A computer program product as set forth in claim 2, wherein the computer program product is tailored for determining the electromagnetic scattering from a time-harmonic incident field in the form of a radar wavefront and where the three-dimensional object represents a component of a vehicle.

4. A computer program product, as set forth in claim 1, wherein the BIE produced is in a form of:

$$(\mathbf{E}\text{ excitation term}) + \gamma(\mathbf{H}\text{ excitation term}) = \{(\mathbf{E}\text{ integral operator term}) + \gamma(\mathbf{H}\text{ integral operation term})\}\mathbf{J}$$

where,

γ is a complex constant,

$\mathbf{J}$ represents the unknown surface source distribution, and

the $(\mathbf{E}\text{ excitation term})$ and $(\mathbf{E}\text{ integral operator term})$ are of a formulation selected from the group consisting of A and B, with

A representing a combination of

$(\mathbf{E}\text{ excitation term}) \equiv T(k_1)\left(-\mathbf{n}(\mathbf{x}) \times \mathbf{E}^{inc}(\mathbf{x})\right)$ and

$(\mathbf{E}\text{ integral operator term}) \equiv T(k_1)\,T(k_2)$, and with

B representing a combination of

$(\mathbf{E}\text{ excitation term}) \equiv$

$ik_1 T^S(k_1)(-\mathbf{n}(\mathbf{x}) \times \mathbf{E}^{inc}(\mathbf{x})) - \zeta \frac{k_2}{k_1} T^a(k_1)\left(Z\mathbf{n}(\mathbf{x}) \cdot \mathbf{H}^{inc}(\mathbf{x})\right)$ and

$(\mathbf{E}\text{ integral operator term}) \equiv$

$$\zeta\frac{k_2}{k_1}T^{\alpha}(k_1)\circ T^{T}(k_2)+\frac{k_1}{k_2}T^{\beta}(k_2)\circ T^{L}(k_2)-k_1k_2T^{S}(k_1)\circ T^{S}(k_2)\text{; and}$$

where the $(\mathbf{H}\text{ excitation term})$ and $(\mathbf{H}\text{ integral operation term})$ are of a formulation selected from the group consisting of A', B', and C', with

A' representing a combination of

$$(\mathbf{H}\text{ excitation term})\equiv Z\mathbf{n}(\mathbf{x})\times\mathbf{H}^{inc}(\mathbf{x})\text{ and}$$

$$(\mathbf{H}\text{ integral operation term})\equiv K^{+}(k_2)\text{;}$$

B' representing a combination of

$$(\mathbf{H}\text{ excitation term})\equiv K^{+}(k_1)\left(Z\mathbf{n}(\mathbf{x})\times\mathbf{H}^{inc}(\mathbf{x})\right)\text{ and}$$

$$(\mathbf{H}\text{ integral operation term})\equiv K^{+}(k_1)\circ K^{+}(k_2)\text{; and}$$

C' representing a combination of

$$(\mathbf{H}\text{ excitation term})\equiv \mathbf{n}\times K^{+}(k_1)\left(Z\mathbf{n}(\mathbf{x})\times\mathbf{H}^{inc}(\mathbf{x})\right)\text{ and}$$

$$(\mathbf{H}\text{ integral operation term})\equiv \left[\mathbf{n}\times K^{+}(k_1)\right]\circ\left[\mathbf{n}\times K^{+}(k_2)\right],$$

wherein;

$$T(k)\equiv ik\mathbf{n}(\mathbf{x})\times\int_S ds'\left\{G(k;\mathbf{x},\mathbf{x}')\mathbf{J}(\mathbf{x})+\frac{1}{k^2}\nabla\left[\nabla G(k;\mathbf{x},\mathbf{x}')\cdot\mathbf{J}(\mathbf{x}')\right]\right\},$$

$$K^{+}(k)\equiv\frac{1}{2}+K(k)\text{, and}$$

$$K(k)\equiv-\mathbf{n}(\mathbf{x})\times\int_S ds'\nabla G(k;\mathbf{x},\mathbf{x}')\times\mathbf{J}(\mathbf{x}')\text{, and}$$

wherein $\mathbf{J}\equiv Z\mathbf{n}\times\mathbf{H}$ is the unknown surface current;

ζ is a complex number;

$\mathbf{E}^{inc}$ and $\mathbf{H}^{inc}$ are incident electric and magnetic fields, respectively;

$Z = \sqrt{\mu/\varepsilon}$ is the wave impedance;

$G(k;\, \mathbf{x}, \mathbf{x}') = \exp(ikr)/4\pi r$ is a 3d Helmholtz kernel,

where $r = |\mathbf{x} - \mathbf{x}'|$ is a distance separating field and source points with harmonic time dependence $e^{-i\omega t}$ assumed, and where

$$T^{\alpha}(k)\phi \equiv \mathbf{n}(\mathbf{x}) \times \int_S ds' \nabla G(k;\, \mathbf{x}, \mathbf{x}')\phi(\mathbf{x}'),$$

$$T^{\beta}(k)\phi \equiv \mathbf{n}(\mathbf{x}) \times \int_S ds' \mathbf{n}(\mathbf{x}') \times \nabla' G(k;\, \mathbf{x}, \mathbf{x}')\phi(\mathbf{x}'),$$

$$T^{L}(k)\mathbf{f} \equiv \int_S ds' \nabla G(k;\, \mathbf{x}, \mathbf{x}')\mathbf{f}(\mathbf{x}'),$$

$$T^{T}(k)\mathbf{f} \equiv \mathbf{n}(\mathbf{x}) \cdot \int_S ds' \nabla G(k;\, \mathbf{x}, \mathbf{x}') \times \mathbf{f}(\mathbf{x}'), \text{ and}$$

$$T^{S}(k)\mathbf{f} \equiv \mathbf{n}(\mathbf{x}) \times \int_S ds' G(k;\, \mathbf{x}, \mathbf{x}')\mathbf{f}(\mathbf{x}'), \text{ and where}$$

∇ and ∇' denote differentiation with respect to the field point $\mathbf{x}$ and source point $\mathbf{x}'$, respectively, and $\mathbf{n}(\mathbf{x})$ and $\mathbf{n}(\mathbf{x}')$ are outward-pointing unit vectors normal to the target surface S at $\mathbf{x}$ and $\mathbf{x}'$, respectively, and

$i \equiv \sqrt{-1}$;

k_2 is the propagation constant in a medium external to S;

k_1 is a complex number chosen to avoid resonances.

5. A computer program product as set forth in claim 4, wherein the computer program product further comprises means, recorded on the recording medium for

modeling electromagnetic scattering from the three-dimensional object due to the time-harmonic incident field as determined from the equivalent surface sources.

6. A computer program product as set forth in claim 4, wherein the BIE produced is in a form of:

$$kT^S(ik)\mathbf{n}\times\mathbf{E}^{inc}+iZT^{\alpha}(ik)\mathbf{n}\cdot\mathbf{H}^{inc}=\left\{i\left[-T^{\alpha}(ik)\circ T^{T}(k)+T^{\beta}(ik)\circ T^{L}(k)-k^2T^{S}(ik)\circ T^{S}(k)\right]-K^{+}(k)\right\}\mathbf{J}$$

and wherein the terms of (ik) are developed by substituting (ik) for (k) in the terms of the formulation set forth in claim 4.

7. A computer program product as set forth in claim 6, wherein the computer program product further comprises means, recorded on the recording medium for modeling electromagnetic scattering from the three-dimensional object due to the time-harmonic incident field as determined from the equivalent surface sources.

8. An apparatus for modeling the electromagnetic response of an arbitrarily shaped three-dimensional object to an arbitrary time-harmonic incident field by means of a well-conditioned boundary integral equation (BIE), the apparatus comprising:

   a. a computer system having a processor, a memory connected with the processor, and an input/output device connected with the processor for receiving parameters representing an arbitrarily shaped three-dimensional object and an arbitrary time-harmonic incident field;

b. means, operating in the processor and the memory of the computer for utilizing the parameters representing the arbitrarily shaped three-dimensional object and the arbitrary time-harmonic incident field for producing a discretized representation of a well-conditioned BIE to provide a well-conditioned finite dimensional linear system whose solution approximates a distribution of equivalent surface sources on a model caused by the time-harmonic incident field; and

c. means, operating in the processor and the memory of the computer for solving the linear system to determine values of the equivalent surface sources; whereby solution of the well-conditioned finite dimensional linear system provides improved computational efficiency and solution accuracy.

9. An apparatus as set forth in claim 8, wherein the apparatus further comprises means, operating in the processor and the memory of the computer for modeling electromagnetic scattering from the three-dimensional object due to the time-harmonic incident field as determined from the equivalent surface sources.

10. An apparatus as set forth in claim 9, wherein the apparatus is tailored for determining the electromagnetic scattering from a time-harmonic incident field represented in the form of a radar wavefront and where the three-dimensional object represents a component of a vehicle.

11. An apparatus as set forth in claim 8, wherein the BIE produced is in a form of:

$$(\mathbf{E}\text{ excitation term}) + \gamma(\mathbf{H}\text{ excitation term}) = \{(\mathbf{E}\text{ integral operator term}) + \gamma(\mathbf{H}\text{ integral operation term})\}\mathbf{J}$$

where,

γ is a complex constant,

J represents an unknown surface source distribution, and

the $(\mathbf{E}\text{ excitation term})$ and $(\mathbf{E}\text{ integral operator term})$ are of a formulation selected from the group consisting of A and B, with

A representing a combination of

$(\mathbf{E}\text{ excitation term}) \equiv T(k_1)\left(-\mathbf{n}(\mathbf{x}) \times \mathbf{E}^{inc}(\mathbf{x})\right)$ and

$(\mathbf{E}\text{ integral operator term}) \equiv T(k_1)\,T(k_2)$, and with

B representing a combination of

$(\mathbf{E}\text{ excitation term}) \equiv$

$$ik_1 T^S(k_1)(-\mathbf{n}(\mathbf{x}) \times \mathbf{E}^{inc}(\mathbf{x})) - \zeta\frac{k_2}{k_1}T^{\alpha}(k_1)\left(Z\mathbf{n}(\mathbf{x}) \cdot \mathbf{H}^{inc}(\mathbf{x})\right) \text{ and}$$

$(\mathbf{E}\text{ integral operator term}) \equiv$

$$\zeta\frac{k_2}{k_1}T^{\alpha}(k_1) \circ T^{T}(k_2) + \frac{k_1}{k_2}T^{\beta}(k_2) \circ T^{L}(k_2) - k_1 k_2 T^S(k_1) \circ T^S(k_2)\text{; and}$$

where the $(\mathbf{H}\text{ excitation term})$ and $(\mathbf{H}\text{ integral operation term})$ are of a formulation selected from the group consisting of A', B', and C', with

A' representing a combination of

$(\mathbf{H}\text{ excitation term}) \equiv Z\mathbf{n}(\mathbf{x}) \times \mathbf{H}^{inc}(\mathbf{x})$ and

$(\mathbf{H}\text{ integral operation term}) \equiv K^{+}(k_2)$;

B' representing a combination of

$$\left(\mathbf{H}\text{ excitation term}\right) \equiv K^{+}(k_1)\left(Z\mathbf{n}(\mathbf{x}) \times \mathbf{H}^{inc}(\mathbf{x})\right) \text{ and}$$

$$\left(\mathbf{H}\text{ integral operation term}\right) \equiv K^{+}(k_1) \circ K^{+}(k_2)\text{; and}$$

C' representing a combination of

$$\left(\mathbf{H}\text{ excitation term}\right) \equiv \mathbf{n} \times K^{+}(k_1)\left(Z\mathbf{n}(\mathbf{x}) \times \mathbf{H}^{inc}(\mathbf{x})\right) \text{ and}$$

$$\left(\mathbf{H}\text{ integral operation term}\right) \equiv \left\lfloor \mathbf{n} \times K^{+}(k_1) \right\rfloor \circ \left\lfloor \mathbf{n} \times K^{+}(k_2) \right\rfloor,$$

wherein;

$$T(k) \equiv ik\mathbf{n}(\mathbf{x}) \times \int_S ds' \left\{ G(k; \mathbf{x}, \mathbf{x}')\mathbf{J}(\mathbf{x}) + \frac{1}{k^2}\nabla\left[\nabla G(k; \mathbf{x}, \mathbf{x}') \cdot \mathbf{J}(\mathbf{x}')\right] \right\},$$

$$K^{+}(k) \equiv \frac{1}{2} + K(k)\text{, and}$$

$$K(k) \equiv -\mathbf{n}(\mathbf{x}) \times \int_S ds' \nabla G(k; \mathbf{x}, \mathbf{x}') \times \mathbf{J}(\mathbf{x}')\text{, and}$$

wherein $\mathbf{J} \equiv Z\mathbf{n} \times \mathbf{H}$ is the unknown surface current;

ζ is a complex number;

$\mathbf{E}^{inc}$ and $\mathbf{H}^{inc}$ are incident electric and magnetic fields, respectively;

$Z = \sqrt{\mu/\varepsilon}$ is a wave impedance;

$G(k; \mathbf{x}, \mathbf{x}') = \exp(ikr)/4\pi r$ is a 3d Helmholtz kernel,

where $r = |\mathbf{x} - \mathbf{x}'|$ is a distance separating field and source points with harmonic time dependence $e^{-i\omega t}$ assumed, and where

$$T^{\alpha}(k)\phi \equiv \mathbf{n}(\mathbf{x}) \times \int_S ds' \nabla G(k; \mathbf{x}, \mathbf{x}')\phi(\mathbf{x}'),$$

$$T^{\beta}(k)\phi \equiv \mathbf{n}(\mathbf{x}) \times \int_S ds' \mathbf{n}(\mathbf{x}') \times \nabla' G(k; \mathbf{x}, \mathbf{x}')\phi(\mathbf{x}'),$$

$$T^L(k)\mathbf{f} \equiv \int_S ds' \nabla G(k;\, \mathbf{x}, \mathbf{x}')\mathbf{f}(\mathbf{x}'),$$

$$T^T(k)\mathbf{f} \equiv \mathbf{n}(\mathbf{x}) \cdot \int_S ds' \nabla G(k;\, \mathbf{x}, \mathbf{x}') \times \mathbf{f}(\mathbf{x}'), \text{ and}$$

$$T^S(k)\mathbf{f} \equiv \mathbf{n}(\mathbf{x}) \times \int_S ds' G(k;\, \mathbf{x}, \mathbf{x}')\mathbf{f}(\mathbf{x}'), \text{ and where}$$

∇ and ∇' denote differentiation with respect to the field point $\mathbf{x}$ and source point $\mathbf{x}'$, respectively, and $\mathbf{n}(\mathbf{x})$ and $\mathbf{n}(\mathbf{x}')$ are outward-pointing unit vectors normal to the target surface S at $\mathbf{x}$ and $\mathbf{x}'$, respectively, and

$i \equiv \sqrt{-1}$;

k_2 is the propagation constant in a medium external to S;

k_1 is a complex number chosen to avoid resonances.

12. An apparatus as set forth in claim 11, wherein the apparatus further comprises means, operating in the processor and the memory of the computer for modeling electromagnetic scattering from the three-dimensional object due to the time-harmonic incident field as determined from the equivalent surface sources.

13. A computer program product as set forth in claim 11, wherein the BIE produced is in a form of:

$$kT^S(ik)\mathbf{n} \times \mathbf{E}^{inc} + iZT^{\alpha}(ik)\mathbf{n} \cdot \mathbf{H}^{inc} = \left\{i\left[-T^{\alpha}(ik) \circ T^T(k) + T^{\beta}(ik) \circ T^L(k) - k^2 T^S(ik) \circ T^S(k)\right] - K^+(k)\right\}\mathbf{J}$$

and wherein the terms of (ik) are developed by substituting (ik) for (k) in the terms of a formulation set forth in claim 11.

14. An apparatus as set forth in claim 13, wherein the apparatus further comprises means, operating in the processor and the memory of the computer for modeling electromagnetic scattering from the three-dimensional object due to the time-harmonic incident field as determined from the equivalent surface sources.

15. A method for modeling an electromagnetic response of an arbitrarily shaped three-dimensional object to an arbitrary time-harmonic incident field by means of a well-conditioned boundary integral equation (BIE), the method operating on a computer system having a processor, a memory connected with the processor, and an input/output device connected with the processor for receiving parameters representing an arbitrarily shaped three-dimensional object and an arbitrary time-harmonic incident field, the method comprising the steps of:

   a. utilizing the parameters representing the arbitrarily shaped three-dimensional object and the arbitrary time-harmonic incident field for producing a discretized representation of a well-conditioned BIE to provide a well-conditioned finite dimensional linear system whose solution approximates a distribution of equivalent surface sources on a model caused by the arbitrary time-harmonic incident field; and
   b. solving the linear system to determine values of the equivalent surface sources; whereby solution of the well-conditioned finite-dimensional linear system provides improved computational efficiency and solution accuracy.

16. A method as set forth in claim 15, wherein the method further comprises the step of modeling electromagnetic scattering from the three-dimensional object due to the time-harmonic incident field as determined from the equivalent surface sources.

17. A method as set forth in claim 16, wherein the time-harmonic incident field is a radar wavefront and wherein the three-dimensional object represents a component of a vehicle, in order to determine the scattering of radar waves from a vehicle component due to in incident radar wave.

18. A method as set forth in claim 15, wherein the BIE produced is in a form of:

$$(\mathbf{E}\text{ excitation term}) + \gamma(\mathbf{H}\text{ excitation term}) = \{(\mathbf{E}\text{ integral operator term}) + \gamma(\mathbf{H}\text{ integral operation term})\}\mathbf{J}$$

where,

γ is a complex,

$\mathbf{J}$ represents an unknown surface source distribution, and

the $(\mathbf{E}\text{ excitation term})$ and $(\mathbf{E}\text{ integral operator term})$ are of a formulation selected from the group consisting of A and B, with

A representing a combination of

$(\mathbf{E}\text{ excitation term}) \equiv T(k_1)\left(-\mathbf{n}(\mathbf{x}) \times \mathbf{E}^{inc}(\mathbf{x})\right)$ and

$(\mathbf{E}\text{ integral operator term}) \equiv T(k_1)\,T(k_2)$, and with

B representing a combination of

$(\mathbf{E}\text{ excitation term}) \equiv$

$$ik_1 T^S(k_1)(-\mathbf{n}(\mathbf{x}) \times \mathbf{E}^{inc}(\mathbf{x})) - \zeta \frac{k_2}{k_1} T^\alpha(k_1)\big(Z\mathbf{n}(\mathbf{x}) \cdot \mathbf{H}^{inc}(\mathbf{x})\big) \text{ and}$$

$\big(\mathbf{E}$ integral operator term$\big) \equiv$

$$\zeta \frac{k_2}{k_1} T^\alpha(k_1) \circ T^T(k_2) + \frac{k_1}{k_2} T^\beta(k_2) \circ T^L(k_2) - k_1 k_2 T^S(k_1) \circ T^S(k_2)\text{; and}$$

where the $\big(\mathbf{H}$ excitation term$\big)$ and $\big(\mathbf{H}$ integral operation term$\big)$ are of a formulation selected from the group consisting of A', B', and C', with

A' representing a combination of

$\big(\mathbf{H}$ excitation term$\big) \equiv Z\mathbf{n}(\mathbf{x}) \times \mathbf{H}^{inc}(\mathbf{x})$ and

$\big(\mathbf{H}$ integral operation term$\big) \equiv K^+(k_2)$;

B' representing a combination of

$\big(\mathbf{H}$ excitation term$\big) \equiv K^+(k_1)\big(Z\mathbf{n}(\mathbf{x}) \times \mathbf{H}^{inc}(\mathbf{x})\big)$ and

$\big(\mathbf{H}$ integral operation term$\big) \equiv K^+(k_1) \circ K^+(k_2)$; and

C' representing a combination of

$\big(\mathbf{H}$ excitation term$\big) \equiv \mathbf{n} \times K^+(k_1)\big(Z\mathbf{n}(\mathbf{x}) \times \mathbf{H}^{inc}(\mathbf{x})\big)$ and

$\big(\mathbf{H}$ integral operation term$\big) \equiv \big[\mathbf{n} \times K^+(k_1)\big] \circ \big[\mathbf{n} \times K^+(k_2)\big]$,

wherein;

$$T(k) \equiv ik\mathbf{n}(\mathbf{x}) \times \int_S ds' \left\{ G(k; \mathbf{x}, \mathbf{x}')\mathbf{J}(\mathbf{x}) + \frac{1}{k^2} \nabla\big[\nabla G(k; \mathbf{x}, \mathbf{x}') \cdot \mathbf{J}(\mathbf{x}')\big] \right\},$$

$$K^+(k) \equiv \frac{1}{2} + K(k)\text{, and}$$

$$K(k) \equiv -\mathbf{n}(\mathbf{x}) \times \int_S ds' \nabla G(k; \mathbf{x}, \mathbf{x}') \times \mathbf{J}(\mathbf{x}')\text{, and}$$

wherein $\mathbf{J} \equiv Z\mathbf{n} \times \mathbf{H}$ is an unknown surface current;

ζ is a complex number;

$\mathbf{E}^{inc}$ and $\mathbf{H}^{inc}$ are incident electric and magnetic fields, respectively;

$Z = \sqrt{\mu/\varepsilon}$ is a wave impedance;

$G(k; \mathbf{x}, \mathbf{x}') = \exp(ikr)/4\pi r$ is a 3d Helmholtz kernel,

where $r = |\mathbf{x} - \mathbf{x}'|$ is a distance separating field and source points with harmonic time dependence $e^{-i\omega t}$ assumed, and where

$$T^{\alpha}(k)\phi \equiv \mathbf{n}(\mathbf{x}) \times \int_S ds' \nabla G(k; \mathbf{x}, \mathbf{x}')\phi(\mathbf{x}'),$$

$$T^{\beta}(k)\phi \equiv \mathbf{n}(\mathbf{x}) \times \int_S ds' \mathbf{n}(\mathbf{x}') \times \nabla' G(k; \mathbf{x}, \mathbf{x}')\phi(\mathbf{x}'),$$

$$T^{L}(k)\mathbf{f} \equiv \int_S ds' \nabla G(k; \mathbf{x}, \mathbf{x}')\mathbf{f}(\mathbf{x}'),$$

$$T^{T}(k)\mathbf{f} \equiv \mathbf{n}(\mathbf{x}) \cdot \int_S ds' \nabla G(k; \mathbf{x}, \mathbf{x}') \times \mathbf{f}(\mathbf{x}'), \text{ and}$$

$$T^{S}(k)\mathbf{f} \equiv \mathbf{n}(\mathbf{x}) \times \int_S ds' G(k; \mathbf{x}, \mathbf{x}')\mathbf{f}(\mathbf{x}'), \text{ and where}$$

∇ and ∇' denote differentiation with respect to the field point $\mathbf{x}$ and source point $\mathbf{x}'$, respectively, and $\mathbf{n}(\mathbf{x})$ and $\mathbf{n}(\mathbf{x}')$ are outward-pointing unit vectors normal to the target surface S at $\mathbf{x}$ and $\mathbf{x}'$, respectively, and

$i \equiv \sqrt{-1}$;

k_2 is the propagation constant in a medium external to S;

k_1 is a complex number chosen to avoid resonances.

19. A method as set forth in claim 18, wherein the method further comprises the step of modeling electromagnetic scattering from the three-dimensional object due to the time-harmonic incident field as determined from the equivalent surface sources.

20. A method as set forth in claim 18, wherein the BIE produced is in the form of:

$$kT^S(ik)\mathbf{n}\times\mathbf{E}^{inc} + iZT^{\alpha}(ik)\mathbf{n}\cdot\mathbf{H}^{inc} = \left\{i\left[-T^{\alpha}(ik)\circ T^{T}(k) + T^{\beta}(ik)\circ T^{L}(k) - k^2T^S(ik)\circ T^S(k)\right] - K^{+}(k)\right\}\mathbf{J}$$

and wherein the terms of (*ik*) are developed by substituting (*ik*) for (*k*) in the terms of the formulation set forth in claim 18.

21. A method as set forth in claim 20, wherein the method further comprises the step of modeling electromagnetic scattering from the three-dimensional object due to the time-harmonic incident field as determined from the equivalent surface sources.